Exercise 1.1 Page: 5

1. Is zero a rational number? Can you write it in the form p/q where p and q are integers and q ≠ 0?

Solution:

We know that a number is said to be rational if it can be written in the form p/q , where p and q are integers and q ≠ 0.

Taking the case of '0',

Zero can be written in the form 0/1, 0/2, 0/3 … as well as , 0/1, 0/2, 0/3 ..

Since it satisfies the necessary condition, we can conclude that 0 can be written in the p/q form, where q can either be positive or negative number.

Hence, 0 is a rational number.

2. Find six rational numbers between 3 and 4.

Solution:

There are infinite rational numbers between 3 and 4.

As we have to find 6 rational numbers between 3 and 4, we will multiply both the numbers, 3 and 4, with 6+1 = 7 (or any number greater than 6)

i.e., 3 × (7/7) = 21/7

and, 4 × (7/7) = 28/7. The numbers in between 21/7 and 28/7 will be rational and will fall between 3 and 4.

Hence, 22/7, 23/7, 24/7, 25/7, 26/7, 27/7 are the 6 rational numbers between 3 and 4.

3. Find five rational numbers between 3/5 and 4/5.

Solution:

There are infinite rational numbers between 3/5 and 4/5.

To find out 5 rational numbers between 3/5 and 4/5, we will multiply both the numbers 3/5 and 4/5

with 5+1=6 (or any number greater than 5)

i.e., (3/5) × (6/6) = 18/30

and, (4/5) × (6/6) = 24/30

The numbers in between18/30 and 24/30 will be rational and will fall between 3/5 and 4/5.

Hence,19/30, 20/30, 21/30, 22/30, 23/30 are the 5 rational numbers between 3/5 and 4/5

4. State whether the following statements are true or false. Give reasons for your answers.

(i) Every natural number is a whole number.

Solution:

True

Natural numbers- Numbers starting from 1 to infinity (without fractions or decimals) i.e., Natural numbers = 1,2,3,4…

Whole numbers – Numbers starting from 0 to infinity (without fractions or decimals)

i.e., Whole numbers = 0,1,2,3…

Or, we can say that whole numbers have all the elements of natural numbers and zero.

Every natural number is a whole number; however, every whole number is not a natural number.

(ii) Every integer is a whole number.

Solution:

False

Integers- Integers are set of numbers that contain positive, negative and 0; excluding fractional and decimal numbers.

i.e., integers= {…-4,-3,-2,-1,0,1,2,3,4…}

Whole numbers- Numbers starting from 0 to infinity (without fractions or decimals)

i.e., Whole numbers= 0,1,2,3….

Hence, we can say that integers include whole numbers as well as negative numbers.

Every whole number is an integer; however, every integer is not a whole number.

(iii) Every rational number is a whole number.

Solution:

False

Rational numbers- All numbers in the form p/q, where p and q are integers and q≠0.

i.e., Rational numbers = 0, 19/30 , 2, 9/-3, -12/7…

Whole numbers- Numbers starting from 0 to infinity (without fractions or decimals)

i.e., Whole numbers= 0,1,2,3….

Hence, we can say that integers include whole numbers as well as negative numbers.

All whole numbers are rational, however, all rational numbers are not whole numbers.

Exercise 1.2 Page: 8

1. State whether the following statements are true or false. Justify your answers.

(i) Every irrational number is a real number.

Solution:

True

Irrational Numbers – A number is said to be irrational, if it **cannot** be written in the p/q, where p and q are integers and $q \neq 0$.

i.e., Irrational numbers = π, e, $\sqrt{3}$, $5+\sqrt{2}$, 6.23146…. , 0.101001001000….

Real numbers – The collection of both rational and irrational numbers are known as real numbers.

i.e., Real numbers = $\sqrt{2}$, $\sqrt{5}$, π, 0.102…

Every irrational number is a real number, however, every real number is not an irrational number.

(ii) Every point on the number line is of the form √m where m is a natural number.

Solution:

False

The statement is false since as per the rule, a negative number cannot be expressed as square roots.

E.g., $\sqrt{9} =3$ is a natural number.

But $\sqrt{2} = 1.414$ is not a natural number.

Similarly, we know that there are negative numbers on the number line, but when we take the root of a negative number it becomes a complex number and not a natural number.

E.g., $\sqrt{-7} = 7i$, where $i = \sqrt{-1}$

The statement that every point on the number line is of the form $\sqrt{m}$, where m is a natural number is false.

(iii) Every real number is an irrational number.

Solution:

False

The statement is false. Real numbers include both irrational and rational numbers. Therefore, every real number cannot be an irrational number.

Real numbers – The collection of both rational and irrational numbers are known as real numbers.

i.e., Real numbers = $\sqrt{2}$, $\sqrt{5}$, , 0.102…

Irrational Numbers – A number is said to be irrational, if it **cannot** be written in the p/q, where p and q are integers and $q \neq 0$.

i.e., Irrational numbers = π, e, $\sqrt{3}$, $5+\sqrt{2}$, 6.23146…. , 0.101001001000….

Every irrational number is a real number, however, every real number is not irrational.

2. Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.

Solution:

No, the square roots of all positive integers are not irrational.

For example,

$\sqrt{4} = 2$ is rational.

$\sqrt{9} = 3$ is rational.

Hence, the square roots of positive integers 4 and 9 are not irrational. (2 and 3, respectively).

3. Show how $\sqrt{5}$ can be represented on the number line.

Solution:

Step 1: Let line AB be of 2 unit on a number line.

Step 2: At B, draw a perpendicular line BC of length 1 unit.

Step 3: Join CA

Step 4: Now, ABC is a right angled triangle. Applying Pythagoras theorem,

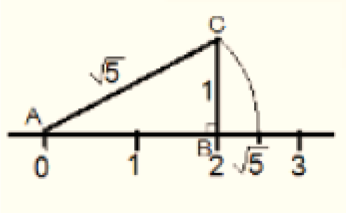
$AB^2+BC^2 = CA^2$

$2^2+1^2 = CA^2 = 5$

$\Rightarrow CA = \sqrt{5}$. Thus, CA is a line of length $\sqrt{5}$ unit.

Step 4: Taking CA as a radius and A as a center draw an arc touching the number line. The point at which number line get intersected by arc is at $\sqrt{5}$ distance from 0 because it is a radius of the circle whose center was A.

Thus, $\sqrt{5}$ is represented on the number line as shown in the figure.



4. Classroom activity (Constructing the 'square root spiral') : Take a large sheet of paper and construct the 'square root spiral' in the following fashion. Start with a point O and draw a line segment OP1 of unit length. Draw a line segment P1P2 perpendicular to OP_1 of unit length (see Fig. 1.9). Now draw a line segment P_2P_3 perpendicular to OP_2. Then draw a line segment P_3P_4 perpendicular to OP_3. Continuing in Fig. 1.9 :

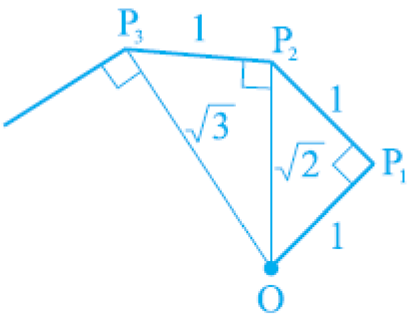
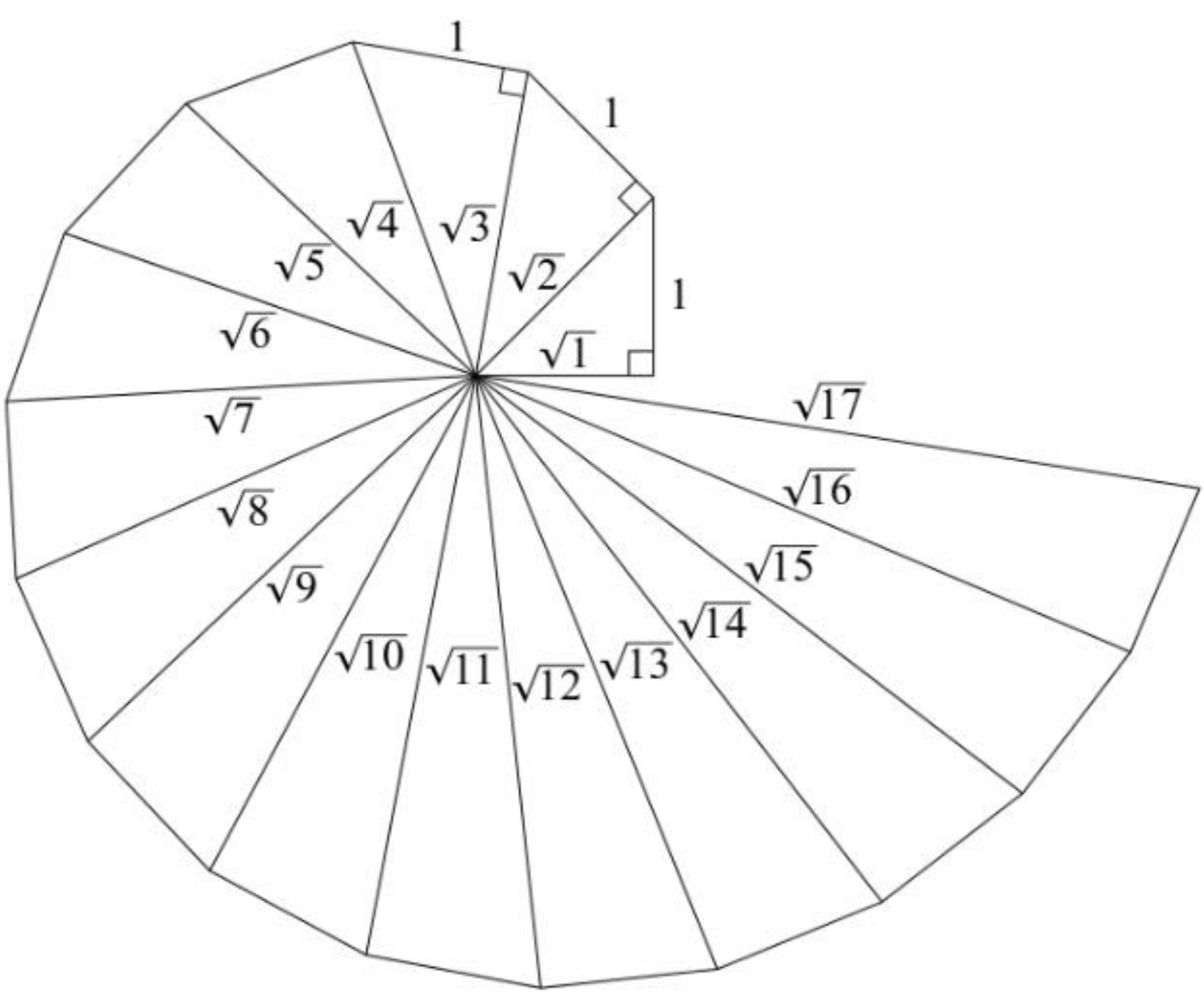


Fig. 1.9 : Constructing square root spiral

Constructing this manner, you can get the line segment $P_{n-1}Pn$ by square root spiral drawing a line segment of unit length perpendicular to OP_{n-1}. In this manner, you will have created the points P_2, P_3,….,Pn,… ., and joined them to create a beautiful spiral depicting √2, √3, √4, …

Solution:



Step 1: Mark a point O on the paper. Here, O will be the center of the square root spiral.

Step 2: From O, draw a straight line, OA, of 1cm horizontally.

Step 3: From A, draw a perpendicular line, AB, of 1 cm.

Step 4: Join OB. Here, OB will be of $\sqrt{2}$

Step 5: Now, from B, draw a perpendicular line of 1 cm and mark the end point C.

Step 6: Join OC. Here, OC will be of $\sqrt{3}$

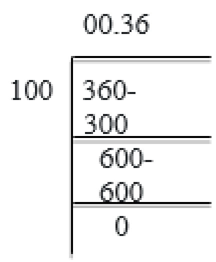
Step 7: Repeat the steps to draw $\sqrt{4}$, $\sqrt{5}$, $\sqrt{6}$….

Exercise 1.3 Page: 14

1. Write the following in decimal form and say what kind of decimal expansion each has :

(i) 36/100

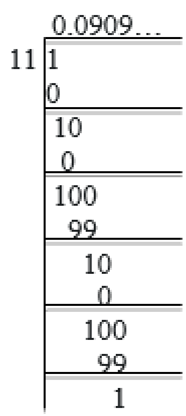
Solution:



= 0.36 (Terminating)

(ii)1/11

Solution:

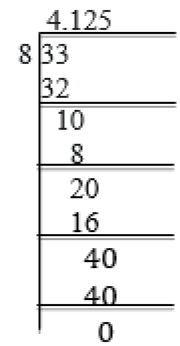


$= 0.0909\ldots = 0.\overline{09}$ (Non terminating and repeating)

(iii) $4\frac{1}{8}$

Solution:

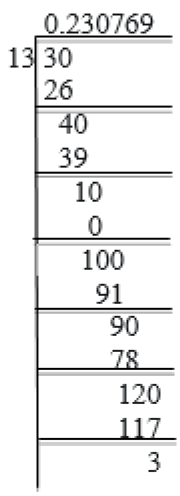
$$4\frac{1}{8} = \frac{33}{8}$$



= 4.125 (Terminating)

(iv) 3/13

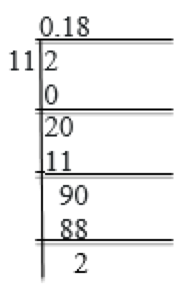
Solution:



$= 0.230769\ldots = 0.\overline{230769}$

(v) 2/11

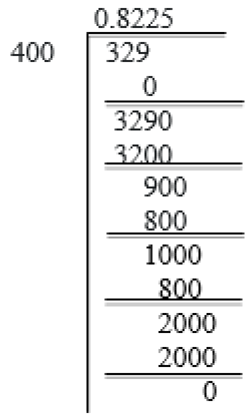
Solution:



$= 0.181818181818\ldots = 0.\overline{18}$ (Non terminating and repeating)

(vi) 329/400

Solution:



= 0.8225 (Terminating)

2. You know that 1/7 = 0.142857. Can you predict what the decimal expansions of 2/7, 3/7, 4/7, 5/7, 6/7 are, without actually doing the long division? If so, how?

[Hint: Study the remainders while finding the value of 1/7 carefully.]

Solution:

$$1/7 = 0.\overline{142857}$$
$$\therefore 2\times 1/7 = 2\times 0.\overline{142857} = 0.\overline{285714}$$
$$3\times 1/7 = 3\times 0.\overline{142857} = 0.\overline{428571}$$
$$4\times 1/7 = 4\times 0.\overline{142857} = 0.\overline{571428}$$
$$5\times 1/7 = 5\times 0.\overline{142857} = 0.\overline{714285}$$
$$6\times 1/7 = 6\times 0.\overline{142857} = 0.\overline{857142}$$

3. Express the following in the form p/q, where p and q are integers and q 0.

(i) $\mathbf{0.\overline{6}}$

Solution:

$0.\overline{6} = 0.666\ldots$

Assume that $x = 0.666\ldots$

Then, $10x = 6.666\ldots$

$10x = 6 + x$

$9x = 6$

$x = 2/3$

(ii)

$0.4\overline{7}$

Solution:

$0.4\overline{7} = 0.4777..$

= (4/10)+(0.777/10)

Assume that x = 0.777…

Then, 10x = 7.777…

10x = 7 + x

x = 7/9

(4/10)+(0.777../10) = (4/10)+(7/90) (x = 7/9 and x = 0.777…0.777…/10 = 7/(9×10) = 7/90)

= (36/90)+(7/90) = 43/90

(iii) $0.\overline{001}$

Solution:

$0.\overline{001} = 0.001001\ldots$

Assume that x = 0.001001…

Then, 1000x = 1.001001…

1000x = 1 + x

999x = 1

x = 1/999

4. Express 0.99999…. in the form p/q . Are you surprised by your answer? With your teacher and classmates discuss why the answer makes sense.

Solution:

Assume that x = 0.9999…..Eq (a)

Multiplying both sides by 10,

10x = 9.9999…. Eq. (b)

Eq.(b) – Eq.(a), we get

10x = 9.9999

–x = -0.9999…

9x = 9

x = 1

The difference between 1 and 0.999999 is 0.000001 which is negligible.

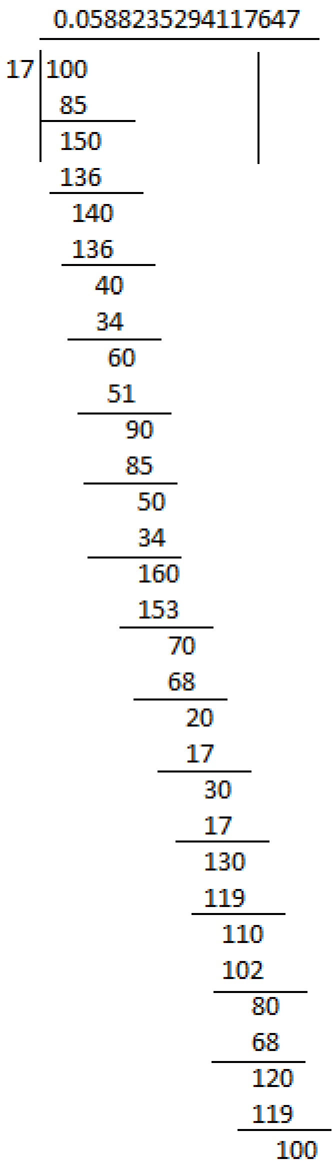
Hence, we can conclude that, 0.999 is too much near 1, therefore, 1 as the answer can be justified.

5. What can the maximum number of digits be in the repeating block of digits in the decimal expansion of 1/17 ? Perform the division to check your answer.

Solution:

1/17

Dividing 1 by 17:



$$\frac{1}{17} = 0.\overline{0588235294117647}$$

There are 16 digits in the repeating block of the decimal expansion of 1/17.

6. Look at several examples of rational numbers in the form p/q (q ≠ 0), where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?

Solution:

We observe that when q is 2, 4, 5, 8, 10… Then the decimal expansion is terminating. For example:

1/2 = 0. 5, denominator q = 2^1

7/8 = 0. 875, denominator q =2^3

4/5 = 0. 8, denominator q = 5^1

We can observe that the terminating decimal may be obtained in the situation where prime factorization of the denominator of the given fractions has the power of only 2 or only 5 or both.

7. Write three numbers whose decimal expansions are non-terminating non-recurring.

Solution:

We know that all irrational numbers are non-terminating non-recurring. three numbers with decimal expansions that are non-terminating non-recurring are:

1. √3 = 1.732050807568
2. √26 =5.099019513592
3. √101 = 10.04987562112

8. Find three different irrational numbers between the rational numbers 5/7 and 9/11.

Solution:

$$\frac{5}{7} = 0.\overline{714285}$$

$$\frac{9}{11} = 0.\overline{81}$$

Three different irrational numbers are:

1. 0.73073007300073000073…
2. 0.75075007300075000075…
3. 0.76076007600076000076…

9. Classify the following numbers as rational or irrational according to their type:

(i)√23

Solution:

√23 = 4.79583152331…

Since the number is non-terminating and non-recurring therefore, it is an irrational number.

(ii)√225

Solution:

$\sqrt{225} = 15 = 15/1$

Since the number can be represented in p/q form, it is a rational number.

(iii) 0.3796

Solution:

Since the number,0.3796, is terminating, it is a rational number.

(iv) 7.478478

Solution:

The number,7.478478, is non-terminating but recurring, it is a rational number.

(v) 1.101001000100001…

Solution:

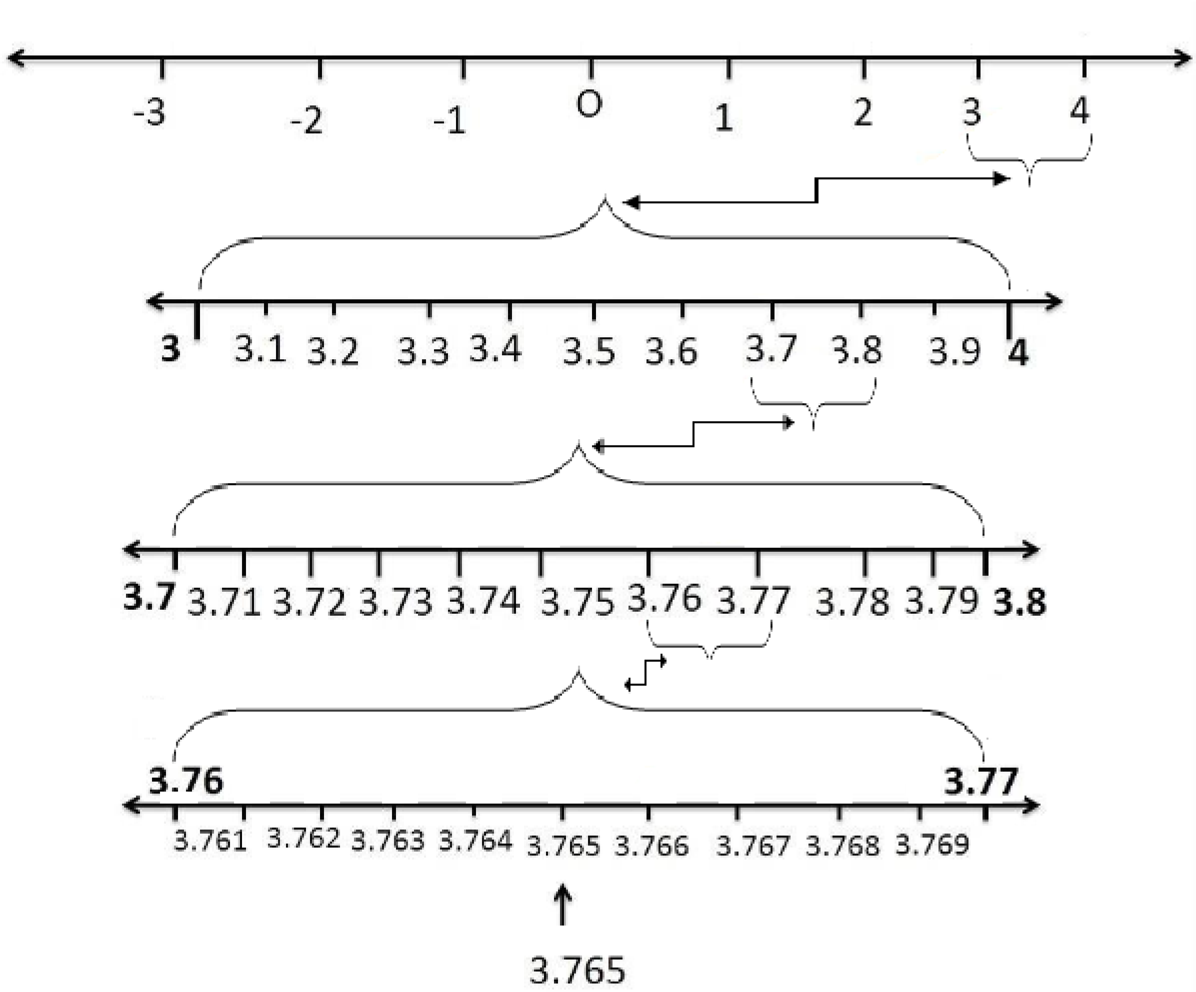
Since the number,1.101001000100001…, is non-terminating non-repeating (non-recurring), it is an irrational number.

Exercise 1.4

Page: 18

1. Visualise 3.765 on the number line, using successive magnification.

Solution:

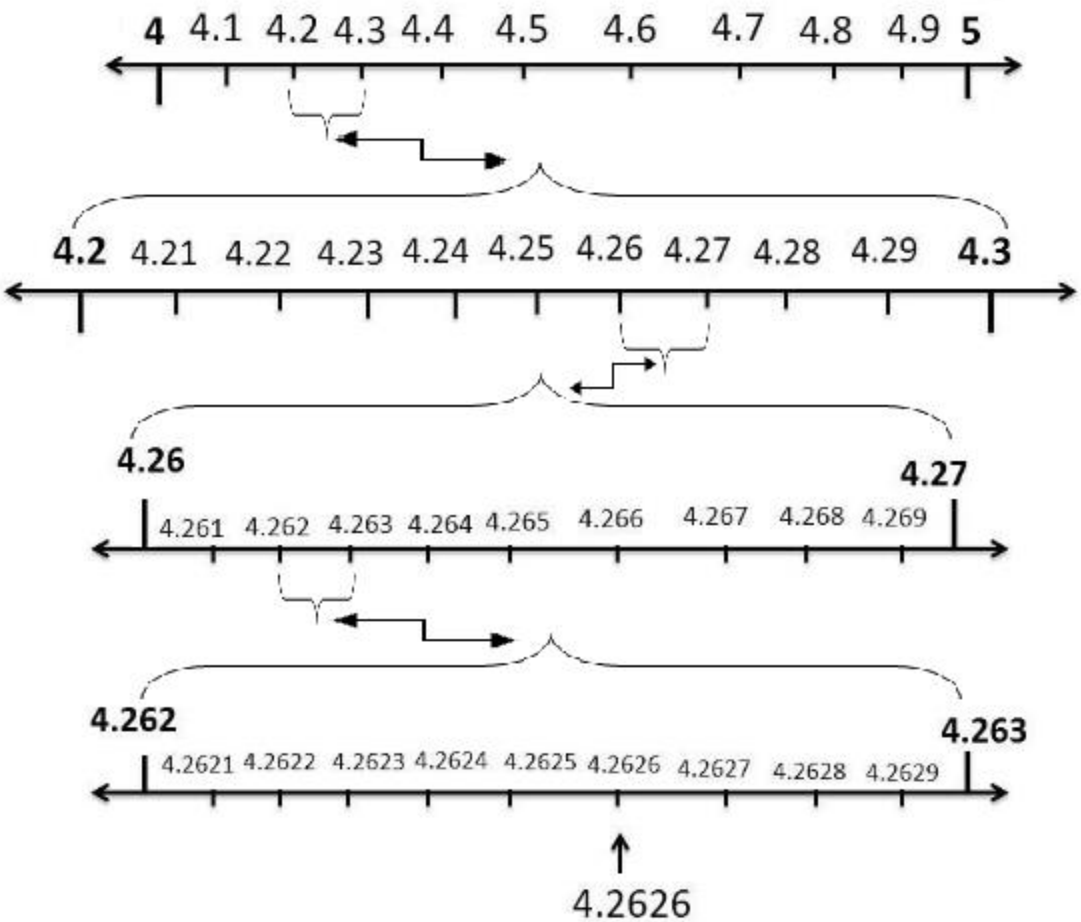


2. Visualise $4.\overline{26}$ on the number line, up to 4 decimal places.

Solution:

$4.\overline{26} = 4.26262626.....$

$4.\overline{26}$ up to 4 decimal places= 4.2626


4 4.1 4.2 4.3 4.4 4.5 4.6 4.7 4.8 4.9 5
4.2 4.21 4.22 4.23 4.24 4.25 4.26 4.27 4.28 4.29 4.3
4.26
4.27
4.261 4.262 4.263 4.264 4.265 4.266 4.267 4.268 4.269
4.262
4.263
4.2621 4.2622 4.2623 4.2624 4.2625 4.2626 4.2627 4.2628 4.2629
4.2626

Exercise 1.5 Page: 24

1. Classify the following numbers as rational or irrational:

(i) $2 - \sqrt{5}$

Solution:

We know that, $\sqrt{5} = 2.2360679\ldots$

Here, 2.2360679…is non-terminating and non-recurring.

Now, substituting the value of $\sqrt{5}$ in $2 - \sqrt{5}$, we get,

$2-\sqrt{5} = 2-2.2360679\ldots = -0.2360679$

Since the number, – 0.2360679…, is non-terminating non-recurring, $2 - \sqrt{5}$ is an irrational number.

(ii) $(3 + \sqrt{23}) - \sqrt{23}$

Solution:

$(3 + \sqrt{23}) - \sqrt{23} = 3 + \sqrt{23} - \sqrt{23}$

$= 3$

$= 3/1$

Since the number 3/1 is in p/q form, **$(3 + \sqrt{23}) - \sqrt{23}$** is rational.

(iii) $2\sqrt{7}/7\sqrt{7}$

Solution:

$2\sqrt{7}/7\sqrt{7} = (2/7) \times (\sqrt{7}/\sqrt{7})$

We know that $(\sqrt{7}/\sqrt{7}) = 1$

Hence, $(2/7) \times (\sqrt{7}/\sqrt{7}) = (2/7) \times 1 = 2/7$

Since the number, 2/7 is in p/q form, $2\sqrt{7}/7\sqrt{7}$ is rational.

(iv) $1/\sqrt{2}$

Solution:

Multiplying and dividing numerator and denominator by $\sqrt{2}$ we get,

$(1/\sqrt{2}) \times (\sqrt{2}/\sqrt{2}) = \sqrt{2}/2$ (since $\sqrt{2} \times \sqrt{2} = 2$)

We know that, $\sqrt{2} = 1.4142\ldots$

Then, $\sqrt{2}/2 = 1.4142/2 = 0.7071..$

Since the number, 0.7071..is non-terminating non-recurring, $1/\sqrt{2}$ is an irrational number.

(v) 2

Solution:

We know that, the value of = 3.1415

Hence, 2 = 2×3.1415.. = 6.2830…

Since the number, 6.2830…, is non-terminating non-recurring, 2 is an irrational number.

2. Simplify each of the following expressions:

(i) $(3+\sqrt{3})(2+\sqrt{2})$

Solution:

$(3+\sqrt{3})(2+\sqrt{2})$

Opening the brackets, we get, $(3\times2)+(3\times\sqrt{2})+(\sqrt{3}\times2)+(\sqrt{3}\times\sqrt{2})$

$= 6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$

(ii) $(3+\sqrt{3})(3-\sqrt{3})$

Solution:

$(3+\sqrt{3})(3-\sqrt{3}) = 3^2-(\sqrt{3})^2 = 9-3$

$= 6$

(iii) $(\sqrt{5}+\sqrt{2})^2$

Solution:

$(\sqrt{5}+\sqrt{2})^2 = \sqrt{5}^2+(2\times\sqrt{5}\times\sqrt{2})+\sqrt{2}^2$

$= 5+2\times\sqrt{10}+2 = 7+2\sqrt{10}$

(iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

Solution:

$(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2}) = (\sqrt{5}^2-\sqrt{2}^2) = 5-2 = 3$

3. Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter, (say d). That is, $\pi = c/d$. This seems to contradict the fact that π is irrational. How will you resolve this contradiction?

Solution:

There is no contradiction. When we measure a value with a scale, we only obtain an approximate value. We never obtain an exact value. Therefore, we may not realize whether c or d is irrational. The value of π is almost equal to 22/7 or 3.142857…

4. Represent $(\sqrt{9.3})$ on the number line.

Solution:

Step 1: Draw a 9.3 units long line segment, AB. Extend AB to C such that BC=1 unit.

Step 2: Now, AC = 10.3 units. Let the centre of AC be O.

Step 3: Draw a semi-circle of radius OC with centre O.

Step 4: Draw a BD perpendicular to AC at point B intersecting the semicircle at D. Join OD.

Step 5: OBD, obtained, is a right angled triangle.

Here, OD 10.3/2 (radius of semi-circle), OC = 10.3/2 , BC = 1

OB = OC – BC

⟹ (10.3/2)-1 = 8.3/2

Using Pythagoras theorem,

We get,

$OD^2=BD^2+OB^2$

⟹ $(10.3/2)^2 = BD^2+(8.3/2)^2$

⟹ $BD^2 = (10.3/2)^2-(8.3/2)^2$

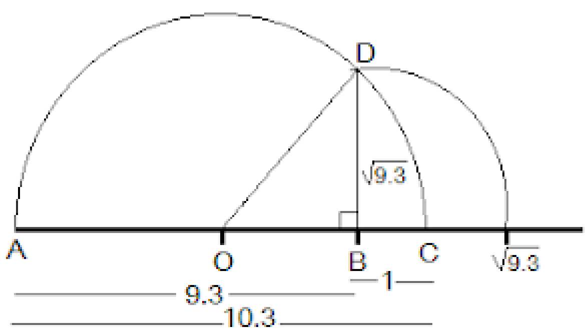
⟹ $(BD)^2 = (10.3/2)-(8.3/2)(10.3/2)+(8.3/2)$

⟹ $BD^2 = 9.3$

⟹ BD = √9.3

Thus, the length of BD is √9.3.

Step 6: Taking BD as radius and B as centre draw an arc which touches the line segment. The point where it touches the line segment is at a distance of √9.3 from O as shown in the figure.



5. Rationalize the denominators of the following:

(i) 1/√7

Solution:

Multiply and divide 1/√7 by √7

(1×√7)/(√7×√7) = √7/7

(ii) 1/(√7-√6)

Solution:

Multiply and divide 1/(√7-√6) by (√7+√6)

[1/(√7-√6)]×(√7+√6)/(√7+√6) = (√7+√6)/(√7-√6)(√7+√6)
= $(\sqrt{7}+\sqrt{6})/\sqrt{7}^2-\sqrt{6}^2$ [denominator is obtained by the property, $(a+b)(a-b) = a^2-b^2$]

= (√7+√6)/(7-6)

= (√7+√6)/1

$= \sqrt{7}+\sqrt{6}$

(iii) $1/(\sqrt{5}+\sqrt{2})$

Solution:

Multiply and divide $1/(\sqrt{5}+\sqrt{2})$ by $(\sqrt{5}-\sqrt{2})$

$[1/(\sqrt{5}+\sqrt{2})]\times(\sqrt{5}-\sqrt{2})/(\sqrt{5}-\sqrt{2}) = (\sqrt{5}-\sqrt{2})/(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})$
$= (\sqrt{5}-\sqrt{2})/(\sqrt{5}^2-\sqrt{2}^2)$ [denominator is obtained by the property, $(a+b)(a-b) = a^2-b^2$]

$= (\sqrt{5}-\sqrt{2})/(5-2)$

$= (\sqrt{5}-\sqrt{2})/3$

(iv) $1/(\sqrt{7}-2)$

Solution:

Multiply and divide $1/(\sqrt{7}-2)$ by $(\sqrt{7}+2)$

$1/(\sqrt{7}-2)\times(\sqrt{7}+2)/(\sqrt{7}+2) = (\sqrt{7}+2)/(\sqrt{7}-2)(\sqrt{7}+2)$

$= (\sqrt{7}+2)/(\sqrt{7}^2-2^2)$ [denominator is obtained by the property, $(a+b)(a-b) = a^2-b^2$]

$= (\sqrt{7}+2)/(7-4)$

$= (\sqrt{7}+2)/3$

Exercise 1.6 Page: 26

1. Find:

(i)$64^{1/2}$

Solution:

$64^{1/2} = (8\times8)^{1/2}$

$= (8^2)^{½}$

$= 8^1$ [$\because 2\times1/2 = 2/2 = 1$]

$= 8$

(ii)$32^{1/5}$

Solution:

$32^{1/5} = (2^5)^{1/5}$

$= (2^5)^{⅕}$

$= 2^1$ [$\because 5\times1/5 = 1$]

$= 2$

(iii)$125^{1/3}$

Solution:

$(125)^{1/3} = (5\times5\times5)^{1/3}$

$= (5^3)^{⅓}$

$= 5^1$ $(3\times1/3 = 3/3 = 1)$

$= 5$

2. Find:

(i) $9^{3/2}$

Solution:

$9^{3/2} = (3\times3)^{3/2}$

$= (3^2)^{3/2}$

$= 3^3$ [$\because 2\times3/2 = 3$]

$=27$

(ii) $32^{2/5}$

Solution:

$32^{2/5} = (2\times2\times2\times2\times2)^{2/5}$

$= (2^5)^{2/5}$

$= 2^2$ [∵5×2/5= 2]

= 4

(iii)$16^{3/4}$

Solution:

$16^{3/4} = (2×2×2×2)^{3/4}$

$= (2^4)^{3/4}$

$= 2^3$ [∵4×3/4 = 3]

= 8

(iv) $125^{-1/3}$

$125^{-1/3} = (5×5×5)^{-1/3}$

$= (5^3)^{-1/3}$

$= 5^{-1}$ [∵3×-1/3 = -1]

= 1/5

3. Simplify:

(i) $2^{2/3}×2^{1/5}$

Solution:

$2^{2/3}×2^{1/5} = 2^{(2/3)+(1/5)}$ [∵Since, $a^m×a^n=a^{m+n}$____ Laws of exponents]

$= 2^{13/15}$ [∵2/3 + 1/5 = (2×5+3×1)/(3×5) = 13/15]

(ii) $(1/3^3)^7$

Solution:

$(1/3^3)^7 = (3^{-3})^7$ [∵Since,$(a^m)^n = a^{m \times n}$____ Laws of exponents]

$= 3^{-21}$

(iii) $11^{1/2}/11^{1/4}$

Solution:

$11^{1/2}/11^{1/4} = 11^{(1/2)-(1/4)}$

$= 11^{1/4}$ [∵(1/2) – (1/4) = (1×4-2×1)/(2×4) = 4-2)/8 = 2/8 = ¼]

(iv) $7^{1/2}×8^{1/2}$

Solution:

$7^{1/2}×8^{1/2} = (7×8)^{1/2}$ [∵Since, $(a^m×b^m = (a×b)^m$ ____ Laws of exponents]

$= 56^{1/2}$